CS 105 – Lab #1 – Binary representation of numbers

Binary numbers are the dough we use to represent all information inside the computer. The purpose of this lab is to practice with some binary number representations. After finishing this lab, you should feel more comfortable expressing numbers in both decimal (base 10) and binary (base 2). We will also look at some simple properties and shortcuts of binary representation.

Part I: Binary to decimal conversion

As an example, let's look at the binary number 11010.

1	**1**	**0**	**1**	**0**
$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

To determine its value in base 10, all we need to do is add up the powers of 2 wherever we see a "1" in our binary number. In this case we have 16 + 8 + 2 = 26.

Practice: Convert each of the following binary numbers into decimal.

Binary	*Which powers of 2 are we adding?*	*Decimal answer*
10101		
11110		
1011		
10110		
101100		

Let's check your answers!
For example, you can go to the Web site wolframalpha.com and enter commands such as "1101 from binary to decimal".

Part II: Decimal to binary conversion: the "Binary Store"

To convert a number n into binary, pretend you have $n to spend at the binary store. Your strategy is to buy the most valuable gifts you can. The price list at the store is simply the powers of two: 1, 2, 4, 8, 16, 32, 64, etc.

Example: Let's say you have $60 to start with.
First, buy a $32 item. This will leave you with 60 – 32 = 28.
Next, buy a $16 item. This leaves you with 28 – 16 = 12.
Next, buy an $8 item. Now you have 12 – 8 = 4.
Finally you can buy a $4 item, and all your money is spent.

Your cashier receipt shows that you bought items costing: 32 + 16 + 8 + 4. Each of these numbers is a power of 2, so we can fill in the blanks. "1" means you bought that item, and "0" means you didn't. In this case, we did not buy any \$2 or \$1 item.

1	**1**	**1**	**1**	**0**	**0**
$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$

Thus, the decimal number 60 is equivalent to the binary 111100.
Try these examples. Convert decimal numbers to binary:

Decimal	*What can you buy at the binary store?*	*Binary answer*
36		
19		
25		
50		
100		

Observations

Looking at the above examples, let's see if we can detect some patterns.

1. Look at binary representations of the odd numbers we did earlier, such as 19, 25, etc. What do they all have in common?

2. Look at the even numbers like 26, 50 and 100. What do their binary representations have in common?

3. Let's do an experiment. We'll start with the binary string 1101.
   a. What number does this equal?

b. Let’s append a 0 onto the end of our original string, to become 11010. What number do we have now?

c. Let’s append a 1 onto the end of our original string, to become 11011. What number do we have now?

d. Let’s come up with a general rule. Let’s say you’ve just written down the binary form of some number x. What will happen to the value of the number if we put a “0” at the end of x? What would happen if we put a “1” at the end?

4. Let’s look at binary strings that have *all 1’s* in them. Determine the values of the following binary numbers:
   a. 11

   b. 111

   c. 1111

   d. 11111

   e. What do all of these numbers (e.g. 1, 11, 111, 1111, 11111, etc.) have in common? Be as specific as you can.

Part III: Binary shorthand

Binary numbers can be painful to write or type when they get long. For example, it turns out that to completely specify a color takes 24 bits! ("bit" = binary digit) Here is an example: 100110010011001111111111

There are 2 kinds of shorthand to help us. They are called octal and hexadecimal.

Octal
Octal means base 8, and 8 is 2^3. Thus, one octal digit can stand for 3 bits.

To avoid confusion, octal numbers can carry a subscript 8 to emphasize that the number is in octal. Thus, 101_8 means the 3-digit octal number 101, which happens to equal 65.

To experiment with octal numbers, let's play with the online calculator again. Set it up so that it will convert from base 8 to base 2. Ask it to do these conversions, and write down the answers:

Octal	*Binary*
4	
7	
5	
475	
547	

In the examples 475 and 547, how many bits are in our answers? ___________________
Can you see the individual octal digits? For example, "475" is "4" followed by "7" followed by "5".

Now let's try a few more octal-to-binary cases:

Octal	*Binary*
6	
1	
2	
612	
126	

How many bits does it take to represent the octal "612"? What about "126"? Why do you think there is a difference?

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Next, set up the online calculator so that it will go the other way: from base 2 to base 8. Ask the calculator to do these conversions:

Binary	*Octal*
110	
110111	
010111000	

Can you see how the octal shorthands are determined? Explain how it is done, in your own words.

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<u>Hexadecimal</u>
The second type of shorthand is called hexadecimal, or just "hex" for short. This means base 16, and 16 = 2^4. Thus, each hex digit stands for <u>4 bits</u>, making hex even more efficient than octal as a shorthand for binary.

To avoid confusion, the subscript 16 sometimes used as an indicator that the number is being written in hexadecimal. Thus, 10_{16} means the 2-digit hexadecimal number 10, which happens to equal sixteen, <u>not</u> the number ten or two!

Set up the online calculator to go from base 16 to base 2. Ask it to do these conversions:

Hex	*Binary*
9	
3	
93	
903	
309	

Again, note the number of bits in our answers.

Now, let's go the other way. Let's ask the online calculator to convert from binary to hexadecimal. Set up the calculator to convert from base 2 to base 16. Ask it to do these conversions. (Note that in some binary strings below, I have inserted a space for readability.)

Binary	*Hex*
0101	
0011	
0011 0101	
1001 0000	
1001 1001	
1100	
1001 1100	

Do the last two results look strange? It turns out that in base 16, we must have 16 different symbols for our digits. The digits 0-9 are already familiar to us, but now we need special symbols to represent digit values 10-15.

To fully understand what is going on with hex shorthand, we can ask the online calculator to convert some numbers from decimal to hex. Write the results in the following table:

Decimal	*Hex*
9	
10	
11	
12	
13	
14	
15	
16	

Decimal	Hex
30	
31	
32	
33	
158	
159	
160	
161	

Let's practice a little more using hexadecimal notation that makes use of letters. Work the following problems, and use the online calculator to check your answers.

5. Convert these binary to hexadecimal:
   a. 101011 (Hint: if the number of bits is not already a multiple of 4, pad with zeros on the left as necessary.)

   b. 1010 0110

   c. 1111 1110 1000 1001 1010

6. Convert these hexadecimal to binary:
   a. 45

   b. a1e

   c. 7f