Number Practice

1. Converting from base 10 into another radix:
   Divide the number repeatedly by the radix and stop when the quotient is zero. The list of remainders in reverse will be your answer.

Let's convert the decimal number 90 into binary, octal and hex:

```
 90 into binary          90 into octal           90 into hex
-----------------       -----------------       ------------------
90 / 2 = 45 rem 0       90 / 8 = 11 rem 2       90 / 16 = 5 rem 10
45 / 2 = 22 rem 1       11 / 8 =  1 rem 3        5 / 16 = 0 rem  5
22 / 2 = 11 rem 0        1 / 8 =  0 rem 1
11 / 2 =  5 rem 1
 5 / 2 =  2 rem 1
 2 / 2 =  1 rem 0
 1 / 2 =  0 rem 1

     1011010                 0132                    0x5a
```

Another example, convert 63.

```
 63 into binary          63 into octal           63 into hex
-----------------       -----------------       ------------------
63 / 2 = 31 rem 1       63 / 8 = 7 rem 7        63 / 16 = 3 rem 15
31 / 2 = 15 rem 1        7 / 8 = 0 rem 7         3 / 16 = 0 rem  3
15 / 2 =  7 rem 1
 7 / 2 =  3 rem 1
 3 / 2 =  1 rem 1
 1 / 2 =  0 rem 1

     111111                  077                     0x3f
```

2. We can reverse these steps to convert back into base 10.

algorithm:

```
read first digit
while (not done)
{
   multiply current number by radix
   add next digit
}
```

As an example, let's translate the octal number 0132 into decimal.

first digit is 1.

multiply 1 by 8 to obtain 8.
add 3 to get 11.

multiply 11 by 8 to obtain 88.
add 2 to get 90.

Another example: let's translate the hex number 0xabf into decimal.

first digit is a = 10.

multiply 10 by 16 to obtain 160.
add b (11) to get 171.

multiply 171 by 16 to obtain 2736.
add f (15) to get 2751.

3. What about digits to the right of the radix point? For a number such as 3.6, we have an integer part (3) to the left of the radix point and a fractional part (6) to the right of the radix point.

To convert a number from base 10 to some other base, the integer part can be translated just as before (successive division). But the fractional part gets translated using successive multiplication, which halts when the fractional part reaches .0 or when we recognize a cycle in our computations.

Let's convert the decimal number 3.6 into binary:

The integer part 3 becomes 11 in binary (since $2^1 + 2^0 = 3$)

The fractional part gets translated using successive multiplication:

.6 * 2 = 1.2	Read the integer parts of these results downward,
.2 * 2 = 0.4	in order, to obtain answer. In this case, 1001 ...
.4 * 2 = 0.8	
.8 * 2 = 1.6	

A cycle will result since we have already encountered ".6".

Thus, .6 becomes .1001 1001 1001 1001 ..., which we express as $.\overline{1001}$

3.6 in base 10 equals $11.\overline{1001}$ in binary.

4. Converting a number between bases that are both powers of two is relatively straightforward. Since $8 = 2^3$, every octal digit corresponds to 3 bits, and since $16 = 2^4$, each hex digit corresponds to 4 bits.

Let's convert the binary number 111010 into octal and hex.

octal: The bits of 111010 can be grouped into 3's: 111 010.
111 = 7 and 010 = 2 in octal, so the answer is 072.

hex: The bits of 111010 can be grouped into 4's: 0011 1010.
0011 = 3 and 1010 = a, so the answer is 0x3a.

As another illustration, let's convert our earlier binary number $11.\overline{1001}$ into octal.

Let's expand this repeating number as $11.\overline{100110011001}$ so that there are 12 bits on the right side of the radix point. Why 12? 12 is a multiple of 3, and we need to group the bits into 3's:

$$011.\overline{100\ 110\ 011\ 001} = 03.\overline{4631}$$

5. As our final example, let's convert this octal value back into decimal. Instead of using the integer converting-to-decimal algorithm as before, we use instead the idea of the weighted positional notation: that each digit corresponds to some power of the radix.

$$03.\overline{4631} = 3 + \left(\frac{4}{8} + \frac{6}{8^2} + \frac{3}{8^3} + \frac{1}{8^4}\right) + \frac{1}{8^4}\left(\frac{4}{8} + \frac{6}{8^2} + \frac{3}{8^3} + \frac{1}{8^4}\right) + \cdots$$

$$= 3 + \frac{4 * 8^3 + 6 * 8^2 + 3 * 8 + 1}{8^4} + \frac{1}{8^4}\left(\frac{4 * 8^3 + 6 * 8^2 + 3 * 8 + 1}{8^4}\right) + \cdots$$

$$= 3 + \frac{2457}{4096} + \frac{1}{4096}\left(\frac{2457}{4096}\right) + \cdots$$

$$= 3 + \frac{\frac{2457}{4096}}{1 - \frac{1}{4096}}$$

$$= 3 + \frac{2457}{4095} = 3 + \frac{819 * 3}{819 * 5} = 3.6$$