

## Using Recursion to Define Sets of Strings

Recursion is exceptionally good at defining sets. We've seen sets of numbers like the Fibonacci sequence. Now let's turn our attention to sets of strings. Once we can define a set of strings, we can do 2 things:

- Generate our own strings from that "language".
- Test a given string to see if it belongs in our set.

In fact, the 2<sup>nd</sup> ability of testing for set membership is what a compiler does when it checks the syntax of your program to see if your text file is a member of the set of legal programs in a given programming language.

Let's do some examples of sets of strings. For each set, we can do the following:

- describe the set in words
- look at typical examples of strings that belong to this set
- write a mathematical-style recursive definition
- write a grammatical-style recursive definition (which is more concise ☺)

In some cases, we can also write a regular expression for the set as well.

1. S = the set of strings containing one or more a's.

Typical examples: a, aa, aaa, aaaa, ...

A recursive definition would be:

'a' is in S.

If the word x is in S, then so is xa.

And the grammar would say:

S → a

S → Sa

2. S = the set of strings that start with an a, followed by zero or more b's.

Typical examples:

a, ab, abb, abbb, abbbb, ...

Recursive definition:

'a' is in S.

If x is in S, then so is xb.

Grammar:

3. S = the set of strings with any number of a's and/or b's

Typical examples:

Recursive definition:

Grammar:

4. S = any number of 0's, followed by any number of 1's

Typical examples:

Recursive definition

Grammar

5.  $S =$  either any number of 0's or any number of 1's (but not both)  
Typical examples:

Recursive definition

Grammar

6.  $S =$  strings with the same number of a's and b's where all the a's come first, followed by all the b's.  
Typical examples:

Recursive definition

Grammar

7.  $S =$  palindromes of odd length (containing only a's and b's)  
Typical examples

Recursive definition

Grammar

8.  $S =$  any palindrome of a's and b's  
Typical examples

Recursive definition

Grammar

9.  $S =$  binary strings that contain two consecutive 1's.  
Typical examples

Recursive definition

Grammar

10.  $S =$  binary strings of odd length whose middle symbol is 0  
Typical examples

Recursive definition

Grammar