$$a_n = 5a_{n-1} - 6a_{n-2} + 2n - 1$$

$$a_n - 5a_{n-1} + 6a_{n-2} = 2n - 1$$ linear poly.

First find the particular solution

$$a_n = an + b$$
$$a_{n-1} = a(n-1) + b$$
$$a_{n-2} = a(n-2) + b$$

$$(an+b) - 5[a(n-1)+b] + 6[a(n-2)+b] = 2n - 1$$

simp, group

$$an + b - 5an + 5a - 5b + 6an - 12a + 6b = 2n - 1$$

$$2an - 7a + 2b = 2n - 1$$ setting the parts equal

$$\begin{cases} 2a = 2 \\ -7a + 2b = -1 \end{cases} \longrightarrow a = 1$$

$$-7 + 2b = -1$$
$$2b = 6$$
$$b = 3$$

particular soln

$$a_n = n + 3$$

Part II: General soln.

$$a_n - 5a_{n-1} + 6a_{n-2} = 2n - 1$$

$$x^2 - 5x + 6 = 0$$
$$(x-2)(x-3) = 0$$
$$x = 2, 3$$

$$a_n = c_1 2^n + c_2 3^n + n + 3$$

Base Cases

$$a_0 = 6$$
$$a_1 = 11$$

if $n=0$: $6 = c_1 + c_2 + 0 + 3$

if $n=1$: $11 = 2c_1 + 3c_2 + 1 + 3$

$$\begin{cases} c_1 + c_2 = 3 \\ 2c_1 + 3c_2 = 7 \end{cases}$$

$$-2c_1 - 2c_2 = -6$$

$$c_2 = 1$$
$$c_1 = 2$$

$$a_n = 2 \cdot 2^n + 3^n + n + 3$$

$$a_n = -a_{n-1} + 12a_{n-2} + 6 \cdot 2^n$$

$$a_n + a_{n-1} - 12a_{n-2} = \underbrace{6 \cdot 2^n}$$

particular soln: $a_n = k \cdot 2^n$

$a_{n-1} = k \cdot 2^{n-1}$

$a_{n-2} = k \cdot 2^{n-2}$

substitute

$$(k \cdot 2^n) + (k \cdot 2^{n-1}) - 12(k \cdot 2^{n-2}) = 6 \cdot 2^n$$

$$k \cdot 2^n + \frac{k}{2} \cdot 2^n - 3k \cdot 2^n = 6 \cdot 2^n$$

$$\left(k + \frac{k}{2} - 3k\right) 2^n = 6 \cdot 2^n$$

$$k + k/2 - 3k = 6$$

$$-\frac{3}{2}k = 6 \longrightarrow k = -4 \longrightarrow$$ particular soln is $a_n = -4 \cdot 2^n$

General solution: of $\underbrace{a_n + a_{n-1} - 12a_{n-2} = 6 \cdot 2^n}$

$x^2 + x - 12 = 0$

$(x-3)(x+4) = 0$

$x = 3, -4$

$$a_n = c_1 3^n + c_2 (-4)^n - 4 \cdot 2^n$$

Base Cases:

$a_0 = 5$

$a_1 = -2$

$\longrightarrow$ if $n=0$: $5 = c_1 + c_2 - 4$

if $n=1$: $-2 = 3c_1 - 4c_2 - 8$

$$\begin{cases} c_1 + c_2 = 9 \\ 3c_1 - 4c_2 = 6 \\ 4c_1 + 4c_2 = 36 \end{cases} \longrightarrow 7c_1 = 42$$

$c_1 = 6$

$c_2 = 3$

$$a_n = 6 \cdot 3^n + 3(-4)^n - 4 \cdot 2^n$$