

$$a_n = 5a_{n-1} - 6a_{n-2} + 2n - 1$$

$$a_n - 5a_{n-1} + 6a_{n-2} = \underline{\underline{2n-1}}$$

linear poly.

First find the particular solution

$$\begin{aligned} a_n &= an + b \\ a_{n-1} &= a(n-1) + b \\ a_{n-2} &= a(n-2) + b \end{aligned}$$

$$(an+b) - 5[a(n-1)+b] + 6[a(n-2)+b] = 2n-1$$

simp, group

$$\underline{an} + b - \underline{5an} + 5a - 5b + \underline{6an} - 12a + 6b = 2n-1$$

$$\underline{2an} - \underline{7a} + \underline{2b} = \underline{2n-1} \quad \text{setting the parts equal}$$

$$\begin{cases} 2a = 2 \\ -7a + 2b = -1 \end{cases} \rightarrow \begin{cases} a = 1 \\ -7 + 2b = -1 \\ 2b = 6 \\ b = 3 \end{cases}$$

particular soln
 $a_n = n + 3$

Part II: General soln.

$$\underline{a_n - 5a_{n-1} + 6a_{n-2}} = \underline{2n-1}$$

$$\begin{aligned} x^2 - 5x + 6 &= 0 \\ (x-2)(x-3) &= 0 \\ x &= 2, 3 \end{aligned}$$

$$a_n = c_1 2^n + c_2 3^n + n + 3$$

Base Cases

$$\begin{aligned} a_0 &= 6 \\ a_1 &= 11 \end{aligned}$$

$$\begin{aligned} \text{if } n=0: & \begin{cases} 6 = c_1 + c_2 + 0 + 3 \end{cases} \\ \text{if } n=1: & \begin{cases} 11 = 2c_1 + 3c_2 + 1 + 3 \end{cases} \end{aligned}$$

$$\begin{cases} c_1 + c_2 = 3 \\ 2c_1 + 3c_2 = 7 \end{cases}$$

$$\begin{aligned} -2c_1 - 2c_2 &= -6 \\ \hline c_2 &= 1 \\ c_1 &= 2 \end{aligned}$$

$$a_n = 2 \cdot 2^n + 3^n + n + 3$$

$$a_n = -a_{n-1} + 12a_{n-2} + 6 \cdot 2^n$$

$$a_n + a_{n-1} - 12a_{n-2} = \underbrace{6 \cdot 2^n}$$



particular soln: $a_n = k \cdot 2^n$
 $a_{n-1} = k \cdot 2^{n-1}$
 $a_{n-2} = k \cdot 2^{n-2}$

substitute

$$(k \cdot 2^n) + (k \cdot 2^{n-1}) - 12(k \cdot 2^{n-2}) = 6 \cdot 2^n$$

$$k \cdot 2^n + \frac{k}{2} \cdot 2^n - 3k \cdot 2^n = 6 \cdot 2^n$$

$$(k + \frac{k}{2} - 3k) 2^n = 6 \cdot 2^n$$

$$k + \frac{k}{2} - 3k = 6$$

$$-\frac{3}{2}k = 6$$

$$\rightarrow k = -4 \rightarrow$$

particular soln is $a_n = -4 \cdot 2^n$

General solution: of $\underbrace{a_n + a_{n-1} - 12a_{n-2} = 6 \cdot 2^n}$

$$x^2 + x - 12 = 0$$

$$(x-3)(x+4) = 0$$

$$x = 3, -4$$

$$a_n = c_1 3^n + c_2 (-4)^n - 4 \cdot 2^n$$

Base Cases:

$$a_0 = 5$$

$$a_1 = -2$$



$$\text{if } n=0 \quad \begin{cases} 5 = c_1 + c_2 - 4 \end{cases}$$

$$\text{if } n=1 \quad \begin{cases} -2 = 3c_1 - 4c_2 - 8 \end{cases}$$

$$\begin{cases} c_1 + c_2 = 9 \end{cases}$$

$$\begin{cases} 3c_1 - 4c_2 = 6 \end{cases}$$

$$4c_1 + 4c_2 = 36$$

$$\left. \begin{matrix} c_1 + c_2 = 9 \\ 3c_1 - 4c_2 = 6 \\ 4c_1 + 4c_2 = 36 \end{matrix} \right\} \rightarrow \begin{matrix} 7c_1 = 42 \\ c_1 = 6 \\ c_2 = 3 \end{matrix}$$

$$\underline{\underline{a_n = 6 \cdot 3^n + 3(-4)^n - 4 \cdot 2^n}}$$