**Excel Solver Tutorial for Optimization**

**What are Solvers Good For?**

Solvers, or optimizers, are software tools that help users find the best way to **allocate scarce resources**. The resources may be raw materials, machine time or people time, money, or anything else in limited supply. The "best" or optimal solution may mean maximizing profits, minimizing costs, or achieving the best possible quality.  An almost infinite variety of problems can be tackled this way, but here are some typical examples:

**Finance and Investment**

* **Working capital management** involves allocating cash to different purposes (accounts receivable, inventory, etc.) across multiple time periods, to maximize interest earnings.
* **Capital budgeting** involves allocating funds to projects that initially consume cash but later generate cash, to maximize a firm's return on capital.
* **Portfolio optimization** -- creating "efficient portfolios" -- involves allocating funds to stocks or bonds to maximize return for a given level of risk, or to minimize risk for a target rate of return.

**Manufacturing**

* **Job shop scheduling** involves allocating time for work orders on different types of production equipment, to minimize delivery time or maximize equipment utilization.
* **Blending** (of petroleum products, ores, animal feed, etc.) involves allocating and combining raw materials of different types and grades, to meet demand while minimizing costs.
* **Cutting stock** (for lumber, paper, etc.) involves allocating space on large sheets or timbers to be cut into smaller pieces, to meet demand while minimizing waste.

**Distribution and Networks**

* **Routing** (of goods, natural gas, electricity, digital data, etc.) involves allocating something to different paths through which it can move to various destinations, to minimize costs or maximize throughput.
* **Loading** (of trucks, rail cars, etc.) involves allocating space in vehicles to items of different sizes so as to minimize wasted or unused space.
* **Scheduling** of everything from workers to vehicles and meeting rooms involves allocating capacity to various tasks in order to meet demand while minimizing overall costs.

**What Must I Do to Use a Solver?**

To use a solver, you must build a **model** that specifies:

* The resources to be used, using **decision variables**,
* The limits on resource usage, called **constraints**, and
* The measure to optimize, called the **objective**.

The solver will find values for the decision variables that satisfy the constraints while optimizing (maximizing or minimizing) the objective.

**Using Spreadsheets.**  Spreadsheets such as Microsoft Excel provide a convenient way to build such a model.  Anyone who has used a spreadsheet is already familiar with the process:  Cells on a worksheet can hold numbers, labels, or formulas that calculate new values -- such as the objective of an optimization.  Constraints are simply limits (specified with <=, = or >= relations) on formula cells.  And the decision variables are simply input cells containing numbers.

**A Sample Model**

Imagine that you are managing a factory that is **building three products**: TV sets, stereos and speakers. Each product is assembled from parts in inventory, and there are five types of parts: Chassis, picture tubes, speaker cones, power supplies and electronics units. Your goal is to produce the mix of products which will maximize profits, given the inventory of products on hand.

From this description, we can see that the **decision variables** are the number of products to build, and the **objective function** will be (gross) profit.  Assume that you can sell TV sets for a gross profit of $75 each, stereos for a profit of $50 each, and speaker cones for $35 each.

To assemble a TV set, you need 1 chassis, 1 picture tube, 2 speaker cones, 1 power supply and 2 sets of electronics. To make a stereo, you need 1 chassis, 2 speaker cones, 1 power supply and 1 set of electronics. And to build a speaker, all you need is 1 speaker cone and 1 set of electronics.  The parts you have on hand are 450 chassis, 250 picture tubes, 800 speaker cones, 450 power supplies and 600 sets of electronics.  This defines the **constraints** in this problem:  You can build only a limited number of products from the parts on hand.

**Writing the Formulas**

Before we implement this problem statement in either Excel or Visual Basic, let's **write out formulas** corresponding to the verbal description above. If we temporarily use the symbol x1 for the number of TV sets assembled, x2 for the number of stereos, and x3 for the number of speakers, our total profit is:

Maximize 75 x1 + 50 x2 + 35 x3 (Profit)

Building each product requires a certain number of parts of each type. For example, TV sets and stereos each require one chassis, but speakers don't use one. The number of parts used depends on the mix of products built (the left hand side of each constraint), and we have a limited number of parts of each type on hand (the corresponding constraint right hand side):

Subject to: (These rules are entered into the solver one at a time.)

1 x1 + 1 x2 + 0 x3 <= 450 (Chassis)
1 x1 + 0 x2 + 0 x3 <= 250 (Picture tubes)
2 x1 + 2 x2 + 1 x3 <= 800 (Speaker cones)
1 x1 + 1 x2 + 0 x3 <= 450 (Power supplies)
2 x1 + 1 x2 + 1 x3 <= 600 (Electronics)

Notice that we've expressed each constraint in a uniform way: Where we aren't using any parts of a particular type, we just use a quantity of 0 parts. This will make it easier to define the constraints in cell ranges in Excel or in arrays in Visual Basic, and calculate their values using a function like SUMPRODUCT in Excel or a simple FOR loop in Visual Basic.

Since the number of products built cannot be negative, we'll also have lower **bounds on the variables**: x1, x2, x3 >= 0.

Now, we'll take you step by step through implementing this Solver model, in either Excel using Premium Solver or in Visual Basic .NET (or another language) using Solver Platform SDK.  Click the link you want to follow first!

**Product Mix Example - in Excel**

**Creating an Excel Worksheet**

Assuming that you have Microsoft Excel, and either the standard Excel Solver or a Premium Solver product installed, the next step is to create a worksheet where the **formulas for the objective function and the constraints** are calculated. In the worksheet below, we have reserved cells D9, E9 and F9 to hold our decision variables *x1*, *x2* and *x3*: The number of TV sets, stereos and speakers to build.  (Click on the worksheet for a full-size image.)



Notice that the gross profit for each assembled product ($75, $50 and $35) is shown in cells D17, E17 and F17. This allows us to compute the objective function in cell D18 as:

=SUMPRODUCT(D17:F17,D9:F9)

In the table area D11:F15, we've entered the number of parts of each type needed to assemble each finished product. For example, the 2 at D13 means that we need 2 speaker cones to build a TV set. These numbers come directly from the formulas for the constraints shown earlier. With these values in place, we can enter a single formula at cell C11:

=SUMPRODUCT(D11:F11,$D$9:$F$9)

to compute the total number of chassis we will use for any given number of TV sets, stereos and speakers. Then we can **copy this formula** to cells C12:C15 to compute the total number of parts used of all the other types. (The dollar signs in $D$9:$F$9 specify that this cell range stays constant, while the cell range D11:F11 becomes D12:F12, D13:F13, etc. in the copied formulas.)

In cells B11:B15, we've entered the number of parts we have in inventory of each type. This allows us to express the constraints shown earlier as:

C11:C15 <= B11:B15

This is a shorthand for C11 <= B11, C12 <= B12, and so on through C15 <= B15. We can enter this form directly in the Solver dialogs. We will also enter the constraint:

D9:F9 >= 0

...to require the decision variables to have non-negative values.

**Using the Solver Dialogs**

To let the Solver know which cells on the worksheet represent the decision variables, constraints and objective function, we click Solver button on the Data tab, or the Premium Solver button on the Add-Ins tab, which displays the Solver Parameters dialog. In the **Set Objective** (or Set Target Cell) edit box, we type or click on cell D18, the objective function. In the **By Changing Variable Cells** edit box, we type D9:F9 or select these cells with the mouse.



To add the constraints, we click on the **Add button**, select cells C11:C15 in the Cell Reference edit box (the *left hand side*), and select cells B11:B15 in the Constraint edit box (the *right hand side*); the default relation <= is OK.  (Click on the dialog to see it full size.)

We choose the Add button again (either from the dialog above, or from the main Solver Parameters dialog) to define the non-negativity constraint on the decision variables.



When we've completely entered the problem, the Solver Parameters dialog looks like this (Click on the dialog to see it full size.)  This is the "traditional" Solver dialog style from Excel 2007 and earlier; the Solver in Excel 2010 has a new style, with similar elements.  Frontline's Premium Solver products can emulate either style, and they also offer a new Ribbon-based user interface.

**Finding and Using the Solution**

To find the optimal solution, we simply click on the **Solve button**. After a moment, the Solver returns the optimal solution: 200 in cells D9 and E9, and 0 in cell F9. This means that we should build 200 TV sets, 200 stereos and no speakers, to earn a total profit of $25,000 (shown at cell D18).



The message "Solver found a solution" appears in the Solver Results dialog, as shown here (Click on the dialog to see it full size).  Again this is the "traditional" Solver dialog style from Excel 2007 and earlier.  We now click on "Answer" in the Reports list box to produce an Answer Report, and click OK to keep the optimal solution values in cells D9:F9.

After a moment, the Solver creates another worksheet containing an **Answer Report**, like the one below, and inserts it to the left of the problem worksheet in the Excel workbook.  (Click on the Answer Report for a full size image.)



This report shows the original and final values of the objective function (target cell) and the decision variables (adjustable cells), as well as the status of each constraint at the optimal solution. Notice that the constraints on speaker cones and electronics are **binding** and have a **slack** value of 0: We used up all of the parts of these two types in building 200 TV sets and 200 stereos, but there were 50 units of each of the other parts left over. If we could obtain additional speaker cones and electronics, we could further increase total profits, but extra units of the other parts would not help in the short run.