Supplement to

Chapter 11

Operational Decision-Making Tools: Transportation and Transshipment Models

In this supplement, you will learn about . . .

- The Transportation Model
- The Transshipment Model

An important factor in supply chain management is determining the lowest-cost transportation provider from among several alternatives. In most cases, items are transported from a plant or warehouse to a producer, a retail outlet, or distributor via truck, rail, or air. Sometimes the modes of transportation may be the same, but the company must decide among different transportation carriers—for example, different trucking firms. Two quantitative techniques that are used for determining the least cost means of transporting goods or services are the *transportation method* and the *transshipment method*.

THE TRANSPORTATION MODEL

• Transportation model: involves transporting items from [illegible]

A **transportation model** is formulated for a class of problems with the following characteristics: (1) a product is *transported* from a number of sources to a number of destinations at the minimum possible cost, and (2) each source is able to supply a fixed number of units of the product and each destination has a fixed demand for the product. The following example demonstrates the formulation of the transportation model.

Example S11.1

A Transportation Problem

Potatoes are grown and harvested on farms in the Midwest and then shipped to distribution centers in Kansas City, Omaha, and Des Moines where they are cleaned and sorted. These distribution centers supply three manufacturing plants operated by the Frodo-Lane Foods Company, located in Chicago, St. Louis, and Cincinnati, where they make potato chips. Potatoes are shipped to the manufacturing plants by railroad or truck. Each distribution center is able to supply the following tons of potatoes to the plants on a monthly basis:

Distribution Center	Supply
1. Kansas City	150
2. Omaha	175
3. Des Moines	275
	600 tons

Each plant demands the following tons of potatoes per month:

Plant	Demand
A. Chicago	200
B. St. Louis	100
C. Cincinnati	300
	600 tons

The cost of transporting 1 ton of potatoes from each distribution center (source) to each plant (destination) differs according to the distance and method of transport. These costs are shown next. For example, the cost of shipping 1 ton of potatoes from the distribution center at Omaha to the plant at Chicago is $7.

	Plant		
Distribution Center	*Chicago*	*St. Louis*	*Cincinnati*
Kansas City	$6	$8	$10
Omaha	7	11	11
Des Moines	4	5	12

The problem is to determine how many tons of potatoes to transport from each distribution center to each plant on a monthly basis to minimize the total cost of transportation. A diagram of the different transportation routes with supply, demand, and cost figures is given in Figure S11.1.

Transportation models are solved within the context of a transportation table, which for our example is shown as follows. Each cell in the table represents the amount transported from one source to one destination. The smaller box within each cell contains the unit transportation cost for that route. For example, in the cell in the upper left-hand corner, the value $6 is the cost of transporting 1 ton of potatoes from Kansas City to Chicago. Along the outer

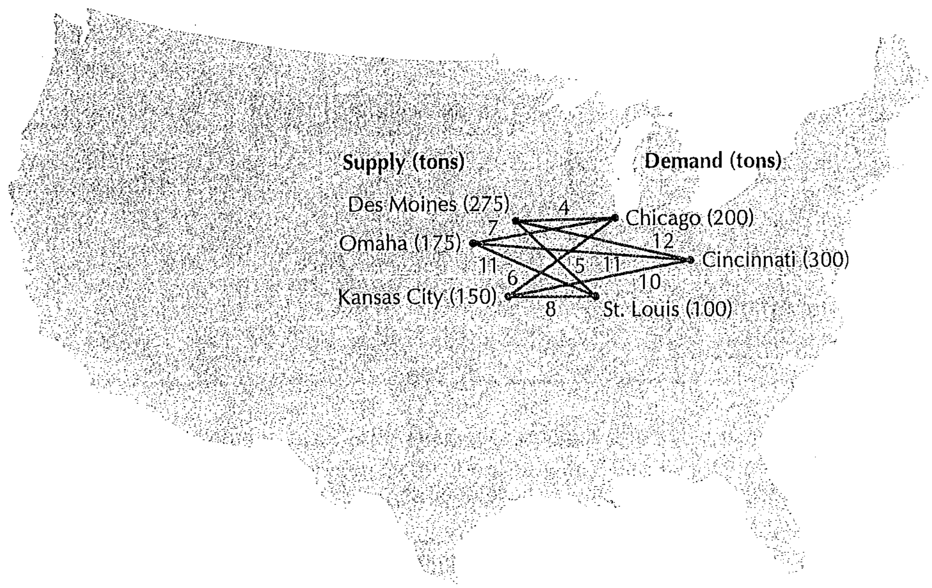


Figure S11.1
Network of Transportation Routes

rim of the table are the supply and demand constraint quantity values, referred to as *rim requirements*.

The Transportation Table

from \ to	*Chicago*	*St. Louis*	*Cincinnati*	**Supply**
Kansas City	6	8	10	150
Omaha	7	11	11	175
Des Moines	4	5	12	275
Demand	200	100	300	600

There are several quantitative methods for solving transportation models manually, including the *stepping-stone method* and the *modified distribution method*. These methods require a number of computational steps and are very time consuming if done by hand. We will not present the detailed solution procedure for these methods here. We will focus on the computer solution of the transportation model using Excel.

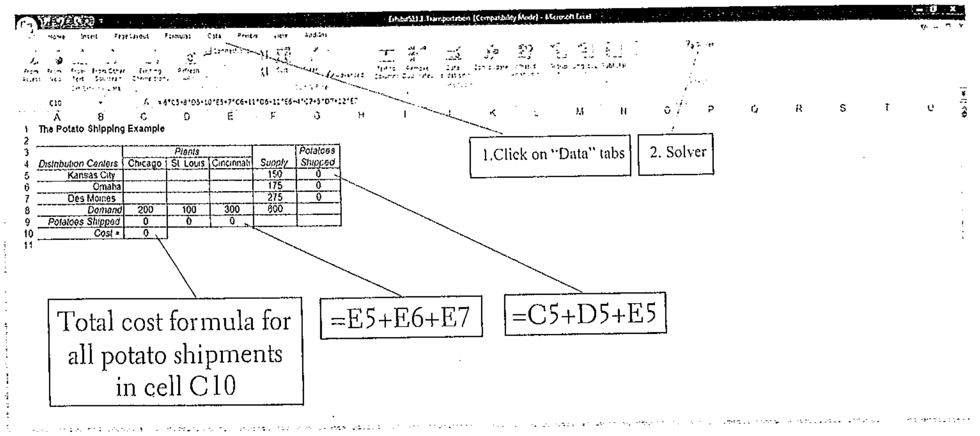
SOLUTION OF THE TRANSPORTATION MODEL WITH EXCEL

Transportation models can be solved using spreadsheets like Microsoft Excel. Exhibit S11.1 shows the initial Excel screen for Example S11.1 (which can be downloaded from the text web site).

Notice in this screen that the formula for the total transportation cost is embedded in cell C10 shown on the formula bar across the top of the screen. Total cost is computed by multiplying each cell cost by each value in cells **C5:E7** inclusive that represent the shipments (currently 0) and summing these products.

Formulas must also be developed for the supply and demand rim requirements. Each distribution center can supply only the amount it has available, and the amount shipped to each plant must

Exhibit S11.1



• Excel File

not exceed what it demands. For example, the amount shipped from Kansas City is the sum of the shipments to Chicago, St. Louis, and Cincinnati.

Similar summation formulas for the other distribution centers and each plant are also developed. If you click on cells G5, G6, G7, C9, D9, and E9, you will see these formulas on the formula bar. Since this is a balanced transportation problem, where total supply equals total demand (i.e., 600 tons), then each amount shipped from each distributor equals the available supply, and each amount shipped to each plant equals the amount demanded. These mathematical relationships are included in the "Solver" screen (shown in Exhibit S11.2) accessed from the "Data" menu on the toolbar.

The "target" cell containing total cost is C10, and it is set equal to "min" since our objective is to minimize cost. The "variables" in our problem representing individual shipments from each distributor to each plant are cells C5 to E7 inclusive. This is designated as "**C5:E7**." (Excel adds the $s.) The constraints mathematically specify that the amount shipped equals the amount available or demanded. For example, "**C9 = C8**" means that the sum of all shipments to Chicago from all three distributors, which is embedded in C9, equals the demand contained in C8. There are six constraints, one for each distributor and plant. There are two more requirements. First, "C5:E7≥0." This specifies that all the amounts shipped must be zero or positive. This can be accomplished by adding this as a constraint, or (as is the case here), clicking on "Options" and then activating the "Assume non-negative" button. Also on the "Options" window activate the "Assume linear models" button. Once all the model parameters have been entered into the solver, click on "Solve." The solution is shown on the Excel screen in Exhibit S11.3.

Interpreting this solution, we find that 125 tons are shipped from Kansas City to Cincinnati, 175 tons are shipped from Omaha to the plant at Cincinnati, and so on. The total shipping cost is $4525. The company could use these results to make decisions about how to ship potatoes and to negotiate new rate agreements with railway and trucking shippers.

Exhibit S11.2

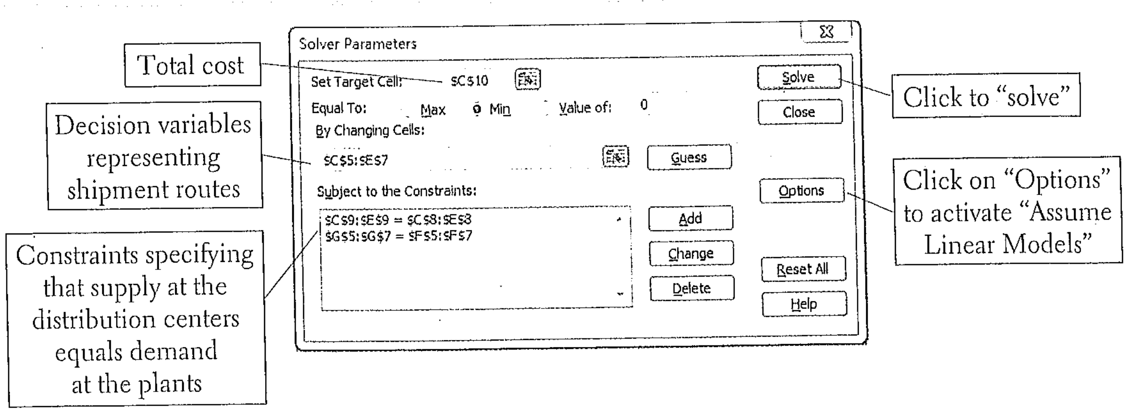
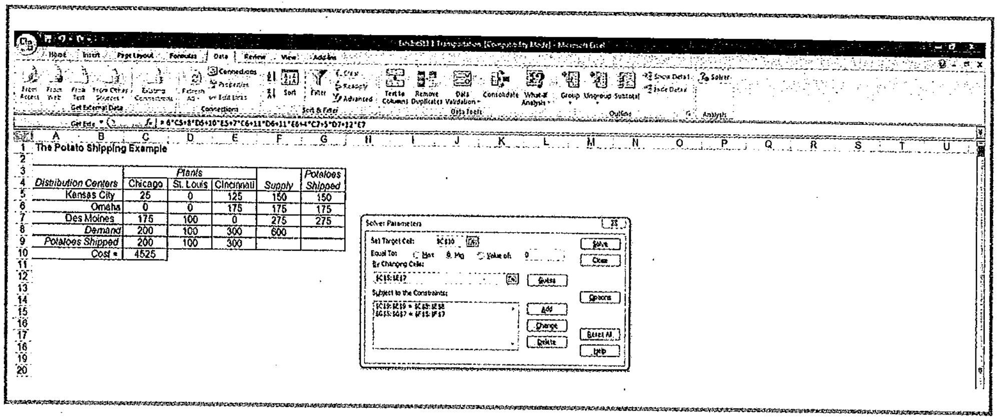


Exhibit S11.3



In this computer solution there is an alternative optimal solution, meaning there is a second solution reflecting a different shipping distribution but with the same total cost of $4525. Manual solution is required to identify this alternative; however, it could provide a different shipping pattern that the company might view as advantageous.

In an *unbalanced transportation problem*, supply exceeds demand or vice versa.

In Example S11.1 the unique condition occurred in which there were the same number of sources as destinations, three, and the supply at all three sources equaled the demand at all three destinations, 600 tons. This is the simplest form of transportation model; however, solution is not restricted to these conditions. Sources and destinations can be unequal, and total supply does not have to equal total demand, which is called an *unbalanced* problem. In addition, there are **prohibited routes.** If a route is prohibited, units cannot be transported from a particular source to a particular destination.

Prohibited route: transportation route over which goods cannot be transported.

Exhibit S11.4 shows the solution for a modified version of our potato shipment example in which supply at Des Moines has been increased to 375 tons and the shipping route from Kansas City to Chicago is prohibited because of a railway track being repaired. An extra column (H) has been added to show the sources that now have excess supply. The cost for cell C5 has been changed from $6 to $100 to prohibit the route from Kansas City to Chicago. The value of $100 is arbitrary; any value can be used that is much larger relative to the other route shipping costs. (Alternatively, this variable, CS, could be eliminated.) Exhibit S11.5 shows the solver for this

Exhibit S11.4

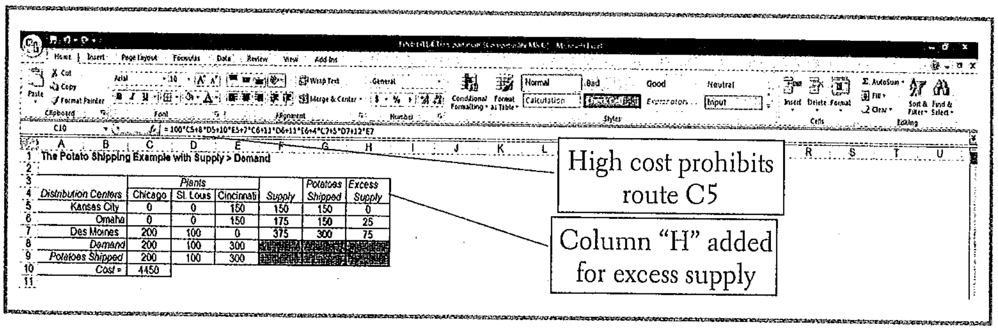


Exhibit S11.5

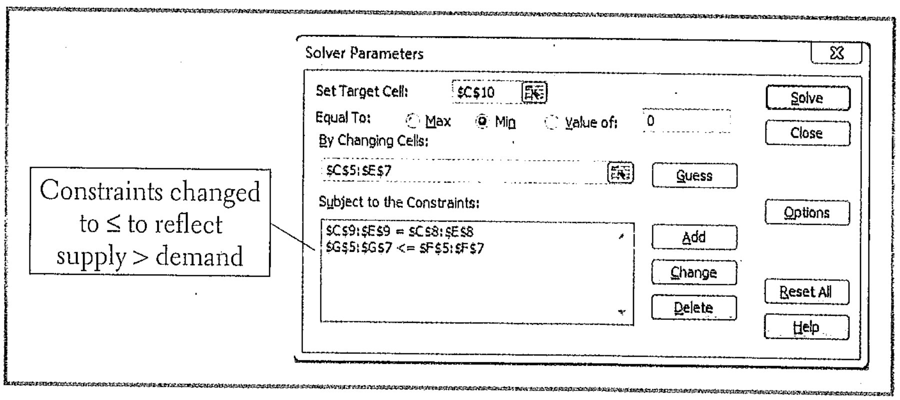
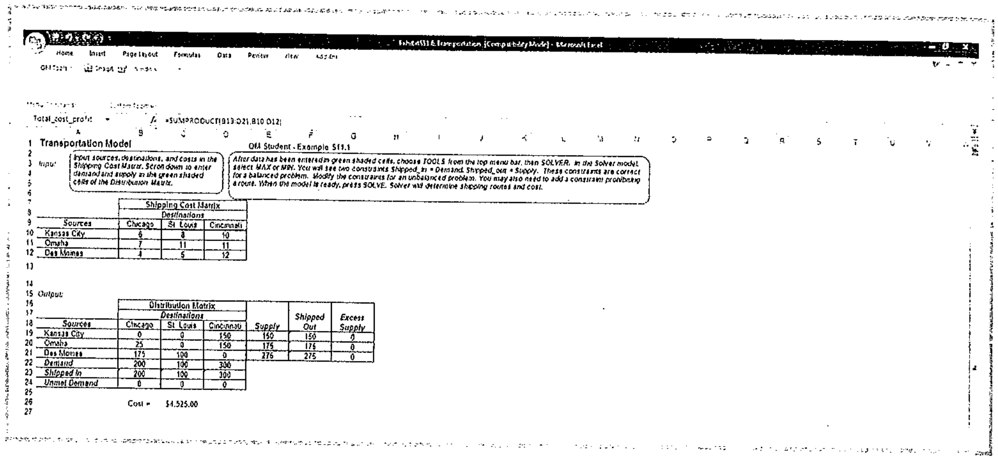


Exhibit S11.6



• OM Tools

problem. The only change in the solver is that the constraints for the potatoes shipped in column "G" are the supply values in column "F."

OM Tools also has a module for solving the transportation model. Exhibit S11.6 shows the OM Tools spreadsheet for Example S11.1.

THE TRANSSHIPMENT MODEL

Transshipment model: a variation of the transportation model with intermediate points between sources and destinations.

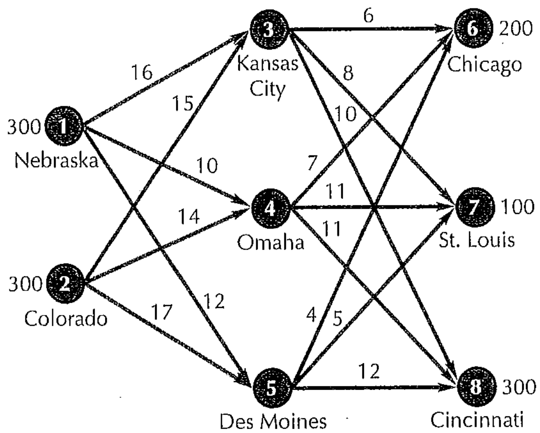
The **transshipment model** is an extension of the transportation model in which intermediate transshipment points are added between the sources and destinations. An example of a transshipment point is a distribution center or warehouse located between plants and stores. In a transshipment problem, items may be transported from sources through transshipment points on to destinations, from one source to another, from one transshipment point to another, from one destination to another, or directly from sources to destinations, or some combination of these alternatives.

Example S11.2
A Transshipment Problem

We will expand our potato shipping example to demonstrate the formulation of a transshipment model. Potatoes are harvested at farms in Nebraska and Colorado before being shipped to the three distribution centers in Kansas City, Omaha, and Des Moines, which are now transshipment points. The amount of potatoes harvested at each farm is 300 tons. The potatoes are then shipped to the plants in Chicago, St. Louis, and Cincinnati. The shipping costs from the distributors to the plants remain the same, and the shipping costs from the farms to the distributors are as follows.

	Distribution Center		
Farm	3. Kansas City	4. Omaha	5. Des Moines
1. Nebraska	$16	10	12
2. Colorado	15	14	17

The basic structure of this model is shown in the following graphical network.



As with the transportation problem, this model includes supply constraints at the farms in Nebraska and Colorado, and demand constraints at the plants in Chicago, St. Louis, and Cincinnati. However, there are several additional mathematical relationships that express the condition that whatever amount is shipped into a distribution center must also be shipped out; that is, the amount shipped into a transshipment point must equal the amount shipped out

SOLUTION OF THE TRANSSHIPMENT PROBLEM WITH EXCEL

Exhibit S11.7 shows the spreadsheet solution and Exhibit S11.8 the Solver for our potato shipping transshipment example. The spreadsheet is similar to the original spreadsheet for the regular transportation problem in Exhibit S11.1, except there are two tables of variables—one for shipping from the farms to the distribution centers and one for shipping potatoes from the distribution centers to the plants. Thus, the decision variables (i.e., the amounts shipped from sources to destinations) are in cells **B6:D7** and **C13:E15**. The constraint for the amount of potatoes shipped from the farm in Nebraska to the three distributors (i.e., the supply constraint for Nebraska) in cell F6 is "**=SUM(B6:D6)**," which sums cells "**B6+C6+D6**." The amount of potatoes shipped to Kansas

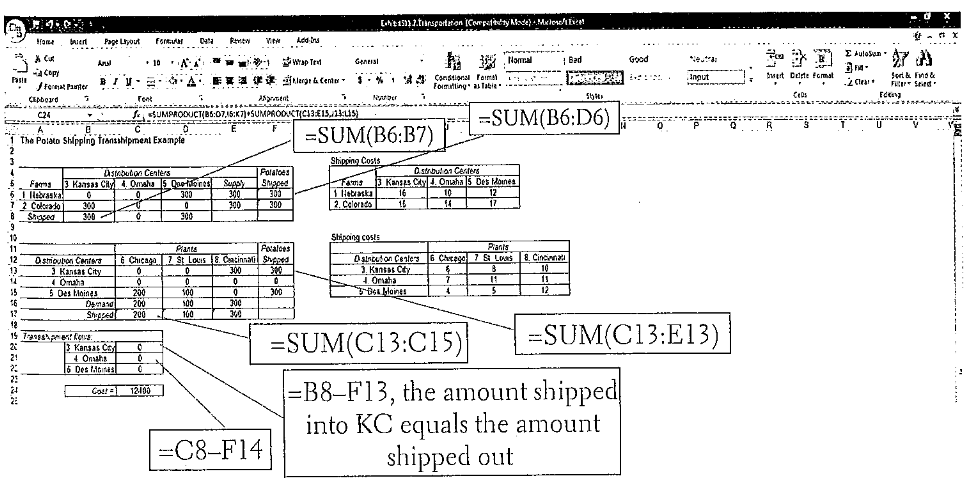
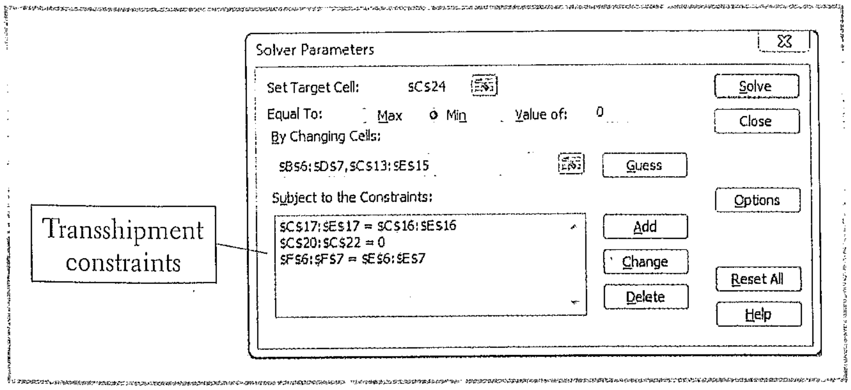


Exhibit S11.7

• Excel File

Exhibit S11.8



City from the farms in cell B8 is "=SUM(B6:B7)." Similar constraints are developed for the sh ments from the distributors to the plants.

The objective function in Exhibit S11.7 is also constructed a bit differently than it was in I hibit S11.1. Instead of typing in a single objective function in cell C24, two cost arrays have b developed for the shipping costs in cells **I6:K7** and cells **J13:L15**, which are then multipl times the variables in cells **B6:D7** and **C13:E15**, and added together. This objective functi "**=SUMPRODUCT(B6:D7,I6:K7)+SUMPRODUCT(C13:E15, J13:L15)**," is shown on toolbar at the top of Exhibit S11.7. Constructing the objective function with cost arrays like thi a little easier than typing in all the variables and costs in a single objective function when there a lot of variables and costs.

OM Tools also has a module for solving the transshipment problem.

SUMMARY

Transportation and transshipment models are quantitative techniques that are used to analyze logistical supply chain problems, specifically the distribution of items from sources to destinations. The objective is frequently to minimize tra portation costs. Both models can be solved using E› spreadsheets which were demonstrated in this chapter.

SUMMARY OF KEY TERMS

prohibited route a transportation route over which items cannot be transported.

transportation model transporting items from sources with fixed supplies to destinations with fixed demands at the minimum cost, time, etc.

transshipment model a special case of the transportation pı lem in which intermediate shipping points exist betw the sources and final destinations.

PLUS

• **Animated Demo Probl**

SOLVED PROBLEMS

TRANSPORTATION MODEL

A manufacturing firm ships its finished products from three plants to three distribution warehouses. The supply capacities of the plants, the demand requirements at the warehouses, and the transportation costs per ton are shown as follows:

Solve this problem using Excel.

SOLUTION

	Warehouses			
PLANT	*A*	*B*	*C*	Supply (units.
1	$8	5	6	120
2	15	10	12	80
3	3	9	10	80
Demand (units)	150	70	60	280

	Distribution Centers				
Inland Port	11. Phoenix	12. Columbus	13. Kansas City	14. Louisville	15. Cleveland
8. Ohio	5	6	5	7	8
9. Texas	6	4	4	5	7
10. North Carolina	10	5	7	4	6

a. Determine the optimal shipping route for each distribution center along this supply chain that will result in the minimum total shipping time. Determine the shipping route and time for each U.S. distributor.

b. Suppose the European ports can only accommodate three shipments each. How will this affect the solution in part (a)?

CASE PROBLEM S11.1

Stateline Shipping and Transport Company

Rachel Sundusky is the manager of the South-Atlantic office of the Stateline Shipping and Transport Company. She is in the process of negotiating a new shipping contract with Polychem, a company that manufactures chemicals for industrial use. Polychem wants Stateline to pick up and transport waste products from its six plants to four waste disposal sites. Rachel is very concerned about this proposed arrangement. The chemical wastes that will be hauled can be hazardous to humans and the environment if they leak. In addition, a number of towns and communities in the region where the plants are located prohibit hazardous materials from being shipped through their municipal limits. Thus, not only will the shipments have to be handled carefully and transported at reduced speeds, they will also have to traverse circuitous routes in many cases.

Rachel has estimated the cost of shipping a barrel of waste from each of the six plants to each of the three waste disposal sites as shown in the following table:

	Waste Disposal Sites		
Plants	Whitewater	Los Canos	Duras
Kingsport	$12	$15	$17
Danville	14	9	10
Macon	13	20	11
Selma	17	16	19
Columbus	7	14	12
Allentown	22	16	18

Each week the plants generate amounts of waste as shown in the following table:

Plant	Waste per Week (BBL)
Kingsport	35
Danville	26
Macon	42
Selma	53
Columbus	29
Allentown	38

The three waste disposal sites at Whitewater, Los Canos, and Duras can accommodate a maximum of 65, 80, and 105 barrels per week, respectively.

The estimated shipping cost per barrel between each of the three waste disposal sites is shown in the following table:

Waste Disposal Site	Whitewater	Los Canos	Duras
Whitewater	—	$12	$10
Los Canos	12	—	15
Duras	10	15	—

In addition to shipping directly from each of the six plants to one of the three waste disposal sites, Rachel is also considering using each of the plants and waste disposal sites as intermediate shipping points. Trucks would be able to drop a load at a plant or disposal site to be picked up and carried on to the final destination by another truck, and vice versa. Stateline would not incur any handling costs since Polychem has agreed to take care of all local handling of the waste materials at the plants and the waste disposal sites. In other words, the only cost Stateline incurs is the actual transportation cost. So Rachel wants to be able to consider the possibility that it may be cheaper to drop and pick up loads at intermediate points rather than shipping them directly.

The following table shows how much Rachel estimates the shipping costs per barrel between each of the six plants to be.

Plants	Kingsport	Danville	Macon	Selma	Columbus	Allentown
Kingsport	—	$6	$1	$9	$7	$8
Danville	6	—	11	10	12	7
Macon	5	11	—	3	7	15
Selma	9	10	3	—	3	16
Columbus	7	12	7	3	—	14
Allentown	8	7	15	16	14	—

Rachel wants to determine the shipping routes that will minimize Stateline's total cost in order to develop a contract proposal to submit to Polychem for waste disposal. She particularly wants to know if it is cheaper to ship directly from the plants to the waste sites or if she should drop and pick up some loads at the various plants and waste sites. Develop a model to assist Rachel and solve the model to determine the optimal routes.

CASE PROBLEM S11.2

Global Supply Chain Management at Cantrex Apparel International

Cantrex Apparel International manufactures clothing items around the world. It has currently contracted with a U.S. retail clothing wholesale distributor for men's goatskin and lambskin leather jackets for the next Christmas season. The distributor has distribution centers in Ohio, Tennessee, and New York. The distributor supplies the leather jackets to a discount retail chain, a chain of mall boutique stores, and a department store chain. The jackets arrive at the distribution centers unfinished, and at the centers the distributor adds a unique lining and label specific to each of its customers. The distributor has contracted with Cantrex to deliver the following number of leather jackets to its distribution centers in late fall:

Distribution Center	Goatskin Jackets	Lambskin Jackets
Ohio	1000	780
Tennessee	1400	950
New York	1600	1150

Cantrex has tanning factories and clothing manufacuring plants to produce leather jackets in Spain, France, :aly, Venezuela, and Brazil. Its tanning facilities are in lende in France, Foggia in Italy, Saragosa in Spain, Feira ı Brazil, and El Tigre in Venezuela. Its manufacturing lants are in Limoges, Naples, Madrid in Europe, and São aulo and Caracas in South America. Following are the upplies of available leather from each tanning facility and the processing capacity at each plant (in lb) for this particular order of leather jackets.

Tanning Factory	Goatskin Supply (lb)	Lambskin Supply (lb)
Mende	4000	4400
Foggia	3700	5300
Saragosa	6500	4650
Feira	5100	6850
El Tigre	3600	5700

Plant	Production Capacity (lb)
Madrid	7800
Naples	5700
Limoges	8200
São Paulo	7600
Caracas	6800

In the production of jackets at the plants 37.5% of the goatskin leather and 50% of the lambskin leather are waste (i.e., it is discarded during the production process and sold for other byproducts). After production, a goatskin jacket weighs approximately 3 lb and a lambskin jacket weighs approximately 2.5 lb (neither with linings which are added in the United States).

Following are the costs/lb (in U.S.$) for tanning the uncut leather, shipping it, and producing the leather jackets at each plant.

(Continued)